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ULTIMATE SLOPES AND DEFLECTIONS -- A BRIEF FOR LIMIT DESIGN

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STRUCTURAL DIVISION

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ULTIMATE SLOPES AND DEFLECTIONS - A BRIEF FOR LIMIT DESIGN

George C. Ernst,[1] M. ASCE

SYNOPSIS

The familiar concepts of slope and deflection are presented as a means of analyzing continuous beams and rigid frames for the evolution of plastic hinges and subsequent failure under static loadings. The presentation may be considered an argument in behalf of limit design, in that the necessary relationship between load and deflection is demonstrated.

INTRODUCTION

Beyond the solely elastic state of stress, adjustments in slopes and deflections are necessary during the development of a number of plastic hinges leading to the ultimate failure of a continuous beam or rigid frame. The necessary adjustments of slopes and deflections, consistent with static equilibrium, may be evaluated in such a manner as to reveal the position and evolution of the plastic hinges by the direct use of revised moment-area theorems, in which the angular yields of plastic hinges are included with a conversion to unit rotations (rates of change of slope) in place of M/EI values.

The conversion to unit rotations is essentially in the use of a unit rotation diagram in place of an M/EI diagram for computations of the elastic portions of slopes and deflections that occur during the yielding of the plastic hinges. A unit rotation diagram, consistent with the character of loading, dimensions of the members, and properties of the materials, may be established for a structure through the fact that all sections other than those yielding as plastic hinges will be stressed below the free yield range. The basic shape of the unit rotation diagram will have the characteristics of a bending moment diagram with all critical sections, consistent with static equilibrium, fully developed. Required bearing lengths and localized failures may be established, as well as the location and evolution of plastic hinges, by the inclusion of the total angular yield of such hinges with the unit rotation diagram.

The letter symbols adopted for use in this paper are defined where they first appear, in the illustrations or in the text, and are arranged alphabetically for convenience of reference in the Appendix.

Revised Moment-Area Theorems

I. The change in slope between any two points on the deflection curve of a flexural member is equal to the sum of the area of the unit rotation diagram and the angular yields of plastic hinges between the corresponding points thereon.

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1. Prof. and Chairman, Dept. of Civ. Eng., Univ. of Nebraska, Lincoln, Nebr.

II. The departure of a point on the axis of an originally straight flexural member, from the tangent to the deflection curve at another point, is equal to the sum of the first or statical moment of the area of the unit rotation diagram and the first moments of the angular yields of plastic hinges between the corresponding points thereon, with respect to the point for which the deflection is sought.

III. The true slope at any point of a flexural member may be calculated as the shear at the same location in a conjugate beam loaded with the unit rotation diagram combined with the angular yields of plastic hinges as concentrated loads located at their respective points of development.

IV. The true deflection at any point of an originally straight flexural member, measured normally from its unloaded axis, is equal to the bending moment at the same location in a conjugate beam loaded with the unit rotation diagram combined with the angular yields of plastic hinges as concentrated loads located at their respective points of development.

Curvature Properties

A short length of flexural member is shown in Figure 1, for which the customary relationship between rate of change of slope is expressed as

$$d\theta = \frac{\varepsilon_c + \varepsilon_t}{d} ds \qquad (1)$$

$$\frac{d\theta}{ds} = \psi = \frac{\varepsilon_c + \varepsilon_t}{d}$$

in which ε_c and ε_t are the maximum unit compressive and tensile strains, d is the depth of the member, and ψ is the angular rotation per unit length of member.

An idealized relationship between the unit rotation, ψ, and $C(M/bd^2)$ is illustrated in Figure 2 with the plastic range included. The constant C is a function of the shape of the cross-section, M is the bending moment, and b is the breadth of the section. A definite limit to the extent of plastic yield is shown, inasmuch as structural materials do not deform without limit. The maximum unit rotation must extend throughout a sufficient length of the flexural member to accumulate the required total angular change at a plastic hinge. In other words, the maximum total angle change at a specific section carrying a constant moment is limited by the distance throughout which the maximum unit rotation extends.

Steel

The plastic properties of structural steel in flexure have been subjected to extensive analytical and experimental research by others.[2,3,4] Such studies

2. "Theory of Inelastic Bending with Reference to Limit Design," A. Hrennikoff, Trans. ASCE, Vol. 113, 1946, p. 213.
3. "Recent Progress in the Plastic Methods of Structural Analysis," Pt. I P. S. Symends and B. G. Neal, Journal of the Franklin Institute, Vol. 252, 1951, p. 387.
4. "Theory of Limit Design" J. A. Van Den Brock, John Wiley & Sons, Inc., New York, New York, 1948.

have yielded broad basic principles for both design and analysis, but have not provided a familiar approach to the more common types of indeterminate structures. Numerical values for Figure 2, for use with the methods of this paper for steel construction, may be obtained from the references cited above.

Reinforced Concrete

For rectangular or teebeams of reinforced concrete, containing less than balanced reinforcement, C = 1. Expressions for unit rotation may be developed in the following manner, for use with Figure 2.

Equating the total compressive force to the total tensile force, and neglecting compression steel at ultimate load,[5]

$$p f_s = k f_{av} \tag{2}$$

in which p is the steel ratio; f_s and f_{av} are steel stress and average concrete stress, respectively; and k is the ratio of distance between extreme fiber and neutral axis to the effective depth of the section.

Also, from the conservation of plane sections

$$\frac{\varepsilon_c}{\varepsilon_t} = \frac{k}{1-k} \tag{3}$$

Solving (2) and (3) to eliminate k

$$\varepsilon_c + \varepsilon_t = \frac{\varepsilon_c f_{av}}{p f_s} \tag{4}$$

For sections on the verge of free yield, and having less than balanced reinforcement

$$\psi_o d = \varepsilon_c + \frac{f_{yp}}{E_s} = \frac{\varepsilon_c f_{av}}{p f_{yp}} \tag{5}$$

And at the ultimate rotational yield, using 0.0038 as the ultimate compressive deformation of all grades of concrete in flexure[6]

$$\psi_u d = 0.0038 + \varepsilon_t = \frac{0.0038 f_{av}}{p f_{yp}} \tag{6}$$

Values from equations (5) and (6) are plotted in Figure 3, using the

5. "Studies in Reinforced Concrete - Section V," Technical Paper No. 22, W. H. Glanville and F. G. Thomas, Dept. of Scientific and Industrial Research, England, 1939.
6. "A Study of Combined Bending and Axial Load in Reinforced Concrete Members," E. Hognestad, Illinois Engrg. Exp. Sta. Bulletin 399, 1951.

Ritter-Hognestad stress-strain curve[6] for three grades of concrete with a steel having a yield point of 40,000 psi. Numerical values taken from Figure 3 must be divided by the effective depth, d, to obtain unit rotation. M/bd^2 may be obtained from a plastic theory equation,[7] or from

$$\frac{M}{bd^2} = j p f_{YP} \qquad (7)$$

in which j is the ratio of the distance between the resultants of the compressive and tensile stresses to the effective depth. Values of j may be obtained from Figure 4.

Unit Rotation, Slope, & Deflection Diagrams

The method proposed for obtaining deflections is essentially that presented many years ago for reinforced concrete beams.[3] The primary difference is in the extension of the method to include ultimate deflections and in the use of the revised moment-area theorems. Figure 5 presents the basic procedure for a beam carrying concentrated loads at the third-points of the span, in which ψ_o represents the unit rotation of the beginning of free yield, and ψ_u is the ultimate unit rotation.

Using a conjugate beam loaded with the unit rotation diagram, the following expressions for deflections are obtained from Figure 5.

$$\text{Deflection at start of free yield} = \frac{23}{216} \psi_o L^2 \qquad (8)$$

$$\text{Ultimate deflection} = \left(\frac{\psi_o}{27} + \frac{5}{72} \psi_u \right) L^2 \qquad (9)$$

Theory vs. Test

Experimental data on the ultimate deflection of beams, particularly reinforced concrete beams, is extremely meager, inasmuch as it is necessary that the deflection be known for which the deformation in the outer fibers has fully developed in both tension and compression. In general, past beam tests, as reported, do not provide such information with any degree of fidelity. However, an outstanding example in reinforced concrete is provided by the test of a full-size precast railway crossing slab, conducted in 1908 by the late Professor A. N. Talbot, Hon. M. ASCE,[9] and for which it was reasonably evident that both concrete and steel had been fully developed without complications due to diagonal tension or bond. A comparison of measured and calculated deflections are provided in Figure 6, from which it may be noted that the agreement is quite good at ultimate value, and does not deviate significantly at lower values. The slab was selected for test from a large number of similar units fabricated

7. "Plastic Theory in Reinforced Concrete Design," C. S. Whitney, Trans. ASCE, Vol. 107, 1942, p. 251.
8. "Relation Between Deflection and Deformation in Reinforced Concrete Beams," G. A. Maney, Proc. ASTM, 1914, p. 310.
9. "A Test of Three Large Reinforced Concrete Beams," A. N. Talbot, Illinois Engrg. Exp. Sta. Bulletin 28, 1908.

by the Illinois Central Railroad for carrying railroad loading. The span was 23' 6", effective depth 30 1/2", breadth 6' 3", steel ratio 0.0127, and weight 33 tons. The beam was tested in the field, with loading continued until compression failure was visually impending in the concrete. The calculated deflections of Figure 6 were obtained through the use of an unidealized M/bd^2 vs. unit rotation curve. The ultimate deflection based upon the idealized form of Figure 2 is given in Table 1, in which several other beams are listed. The agreement between test values and calculated deflections is quite good, in view of the experimental difficulties met with in carrying a beam test to a true flexural ultimate deflection.

Continuous Beams

A two-span continuous beam with an unsymmetrically placed concentrated load on the right hand span is shown in Figure 7a. The location and magnitude of free plastic yielding is desired for the ultimate value of the concentrated load, with the steel ratio equal to 0.01 for both positive and negative moments, f'_c = 4000 psi, f_{yp} = 40,000 psi, and d = 20 inches. From Figure 3, ψ_o = 0.002/20 = 0.0001 radians per inch, and ψ_u = 0.0256/20 = 0.00128 radians per inch.

The unit rotation diagram is shown as a load on the conjugate beam in Figure 7(b), with the critical sections at R2 and under the load fully developed. Prior to solution, the location and magnitude of free plastic yielding is not known but the adjustment to be made by yield would be given by the deflection of the right end assumed to be unsupported. Yielding at R_2 would adjust the right end downward, whereas yielding under the load would move the right end up, for maintaining the positions of the supports. The direction and location of the angular yield, $\psi\lambda$, as shown in Figure 7b, assumes an upward adjustment at the right end and infers that free plastic yielding must occur under the load. It is evident that $\psi\lambda X$ will be negative and yield will occur at R2, if the above assumption is incorrect. In $\psi\lambda X$, λ is the distance throughout which ψ is distributed, and x is the distance from the point of assumed deflection to the section of plastic yielding.

For the conjugate beam

$$\Sigma M_B = 0; \qquad 200\,\theta_L - \frac{(0.0001)(200)^2}{(2)(3)} = 0$$

$$\theta_L = \frac{1}{300}$$

$$\Sigma M_C = 0; \qquad \frac{4}{3} - \frac{8}{3} + \frac{1.12}{3} + \psi\lambda X = 0$$

$$\psi\lambda X = 0.96 \text{ inches}$$

The necessary adjustment in deflection at R3 is 0.96 inches in an upward direction. Since the section under the load is required to make this adjustment by plastic yielding, the position (x) is 120 inches measured from R3. The magnitude of angular yield is equal to 0.96/120 = 0.008 radians under the load. With an ultimate limit of 0.00128 for ψ, the minimum length of beam throughout which the 0.008 radians must be accumulated in order to prevent premature

failure of the section is 0.008/0.00128, or somewhat over six inches. The true slope diagram and deflection curve are shown in Figures 7c and 7d, respectively.

If consecutive pairs of spans are analyzed in the above manner, for a continuous beam of any number of spans, the yielding sections at ultimate load may be determined through the relative magnitudes and directions of the required adjustments in slopes and deflections. The three-span beam of Figure 8 illustrates the case, for which the conjugate beam computations follow. The beam properties are as for the two-span beam.

$$\Sigma_A^B M_B = 0; \qquad \theta = \frac{1}{300}$$

$$\Sigma_A^C M_C = 0; \qquad (\psi \lambda X)_1 = 1.2"$$

The value of 1.2 inches is an upward adjustment required at R3, which locates a yield under the load equal to 1.2/120 = 0.01 radians, which must be included as a concentrated load 120 inches to the left of point C on the conjugate beam in calculating the value of $(\psi \lambda X)_2$ from

$$\Sigma_A^D M_D = 0; \quad (\psi \lambda X)_2 = -\frac{2}{3}"$$

The direction of $(\psi \lambda X)_2$ was assumed incorrectly, therefore the necessary adjustment at R_4 is 2/3 of an inch upwards. This direction of adjustment can only occur through additional plastic yielding under the load combined with plastic yielding at R2. The angular movement from the load to θ_R, necessary for the adjustment of 2/3 of an inch upward at R4, is $(2/3) \div 200 = 1/300$ radians. This drops the beam 120(1/300) = 0.40 inches under the load, thereby rotating the beam from R2 to the load an amount equal to 0.40/80 = 0.005 radians, which is the plastic angular yield required of the section at R2. The total plastic angular yield under the load is the sum of all angular yields accumulating at that section, or 0.0100 + 0.0033 + 0.0050 = 0.0183 radians. This magnitude of yield would require a minimum bearing length of 0.0183 divided by 0.00128, or more than 14 inches, if local distress is to be avoided. The sequence of development of the yielding sections, as identified by relative magnitudes of yield, may be noted to occur under the load first, followed by yielding at R2. A shorter solution, in which the full yield of 0.0183 under the load is obtained in its entirety, is found by proceeding from D to A.

Rigid Frames

A two-hinged rectangular frame, carrying an unsymmetrically placed concentrated vertical load, is shown in Figure 9. Steel ratios, f'_c, f_{yp} and d are the same as for the beams. If the frame is free to side-sway, the solution from conjugate beam analysis will be as outlined below.

$$\Sigma_B^A M_A = 0; \quad \Delta_B = 300\,\theta_B - 3$$

$$\Sigma_C^D M_D = 0; \quad \Delta_C = 3 - 300\,\theta_C$$

$$\Delta_B = \Delta_C; \quad 300\theta_B + 300\theta_C - 6 = 0$$

$$\Sigma V = 0; \quad \theta_B + \theta_C - \psi\lambda = 0$$

$$\Sigma_B^C M_C = 0; \quad 200\,\theta_B - \tfrac{1}{3} - 150\,\psi\lambda = 0$$

Solving the last three equations simultaneously for θ_B , θ_C , and $\psi\lambda$

$$\psi\lambda = 0.02 \text{ radians}$$

$$\theta_B = \tfrac{1}{60} \text{ radians}$$

$$\theta_C = \tfrac{1}{300} \text{ radians}$$

and

$$\Delta_B = \Delta_C = 2 \text{ inches}$$

If side-sway is prevented, the outline of solution by conjugate beam method is

$$\Sigma_D^C M_C = 0; \quad \theta_D = 0.005$$

$$\Sigma_A^C M_C = 0; \quad (\psi\lambda x)_1 = 2\tfrac{1}{3}''$$

$$(\psi\lambda)_1 = \tfrac{7}{150} = 0.0467 \text{ radians at the load.}$$

$$\Sigma_D^A M_A = 0; \quad (\psi\lambda x)_2 = -8'', \text{ to the right of A.}$$

$$(\psi\lambda)_2 = \tfrac{8}{300} = 0.0267 \text{ radians at B.}$$

With the limit of 0.00128 radians per inch as the maximum value of ψ_u , it becomes evident that the length of member throughout which the value of 0.00128 must be distributed in order to fully develop the knee sections is not

a rational possiblity when sidesway is prevented, unless the load is broken into two concentrated loads about three feet apart or distributed by some other means. Inasmuch as there are no means by which distribution of any significant value can be assured at the knee, unless very carefully proportioned, the required length of 0.0267/0.00128 = 21 inches at B would probably prevent the full development of C. Indeed, it is doubtful that the knees for full sidesway would be fully developed unless care was taken to assure a distribution of yielding at the load over a distance of 0.02/0.00128 = 15.6 inches.

The same frame, fixed at A and hinged at D, is illustrated in Figure 10, for which yielding develops first under the load, and then at B, with the following values

$$\psi\lambda \text{ at load} = 0.027 \text{ radians}$$

$$\psi\lambda \text{ at B} = 0.007 \text{ radians}$$

A general solution for the frame of Figure 10 may be developed from Figure 11, resulting in the following groups of equations controlling the bent as the load increases from 0 to the ultimate value of 8000 pounds.

Prior to yielding:

$$15000\psi_A - 37500\psi_B - 52500\psi_C + 30000\psi_E = 0$$

$$90000\psi_A - 103750\psi_B - 11250\psi_C + 35000\psi_E = 0$$

$$\psi_A + \psi_B - \psi_C = 0$$

With yielding at the load only:

$$15000\psi_A - 37500\psi_B - 52500\psi_C + 300(\psi'\lambda)_E = -3$$

$$90000\psi_A - 103750\psi_B - 11250\psi_C + 450(\psi'\lambda)_E = -3.5$$

$$\psi_A + \psi_B - \psi_C = 0$$

With yielding at the load and at B:

$$15000\psi_A - 300(\psi'\lambda)_B - 52500\psi_C + 300(\psi'\lambda)_E = 0.75$$

$$90000\psi_A - 600(\psi'\lambda)_B - 11250\psi_C + 400(\psi'\lambda)_E = 6.875$$

$$\psi_A - \psi_C = -0.0001$$

The solution of these equations throughout loading from zero to ultimate is plotted in Figure 12, from which it may be noted that due to the requirements for maintaining static equilibrium, the knee having the highest moment prior to initial yielding of any section did not develop as rapidly as the other knee after the start of yield at the load.

SUMMARY

It has been demonstrated that ultimate slopes and deflections may be determined in the same basic manner as elastic slopes and deflections, through the use of the curvature properties of the cross-sections in flexure.

Revised moment-area theorems are presented as a means for analyzing continuous beams and rigid frames for the magnitude and location of plastic yielding, the required length of distribution of such yield, and the possibility of local failure prior to full development of all critical sections.

ACKNOWLEDGMENT

The author is indebted to G. R. Swihart, Asst. Prof., Dept. of Civil Engineering, University of Nebraska, for the checking of the continuous beam and rigid frame examples.

APPENDIX NOTATION

The following symbols, adopted for use in this paper and for the guidance of discussers, conform essentially with American Standard Letter Symbols for Structural Analysis (ASA-Z 10.8 - 1949), prepared by a committee of the American Standards Association, with Society representation, and approved by the Association in 1949:

- b = Breadth of flexural member, inches
- C = A constant
- d = Depth of flexural member, as measured from ε_c to ε_t, inches
- E = Modulus of elasticity, psi.
- Es = Modulus of elasticity of steel, psi
- ε_c = Unit compressive strain, inches per inch.
- ε_t = Unit tensile strain, inches per inch
- f'_c = Cylinder strength of concrete, psi.
- f_{av} = Average concrete compressive stress, psi.
- f_s = Steel stress, psi.
- f_{yp} = Yield point stress, psi.
- I = Moment of inertia, in^4.
- j = Ratio of distance between the resultants of the compressive and tensile stresses to the effective depth for a reinforced concrete section.

k = Ratio of distance between extreme fiber and neutral axis to the effective depth of a reinforced concrete section.

L = Span length, inches

M = Bending moment, inch pounds.

λ = Distance throughout which yielding is distributed, inches

p = Steel ratio

s = Distance along beam axis, inches

θ = Beam slope, radians

$\psi = \frac{d\theta}{ds}$, or unit rotation, radians per inch

ψ_o = Unit rotation at start of free yield, radians per inch

ψ_u = Ultimate unit rotation, radians per inch

x = Distance to yielding section, inches

Source	Beam No.	L ft.	b in.	d in.	p	f psi	f' psi	Failure	Deflections (inches)	
									Meas.	Calc.
1	2	3	4	5	6	7	8	9	10	11
Illinois Exp. Sta. Bltn. 1	28	14	12	12	.0152	57,500	1800	C	1.35	1.35
" " " " 1	29	10.7	12	6.5	.0148	55,000	1800	C	1.52	1.51
" " " " 4	35	12	8	10	.0148	33,000	1500	T	1.60	1.17
" " " " 4	37	12	8	10	.0124	33,000	1500	T	1.80	1.54
" " " " 28	72	23.5	75	30.5	.0127	52,000	4000*	T	3.50	3.08
NBS – TP – 2	491	12	8	9.3	.0172	37,900	1589	T	1.12	1.26

*Estimated

TABLE 1

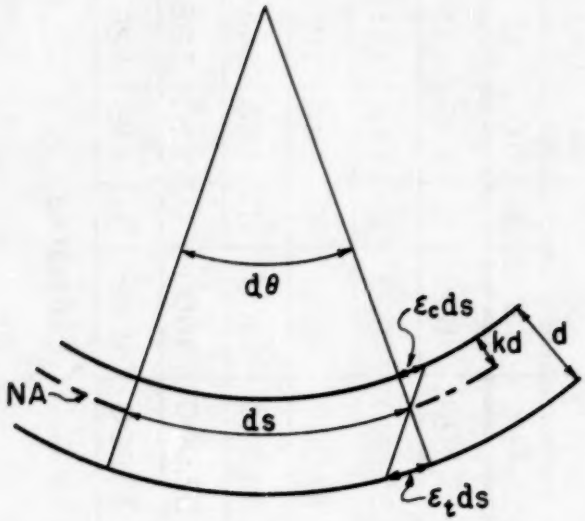


FIGURE 1

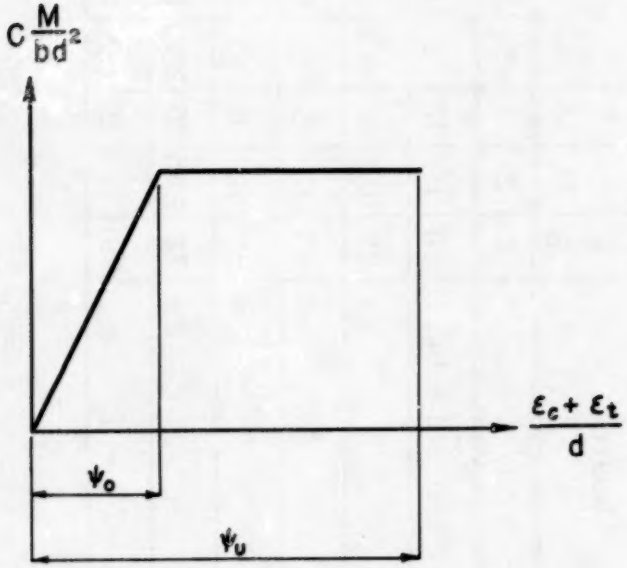


FIGURE 2

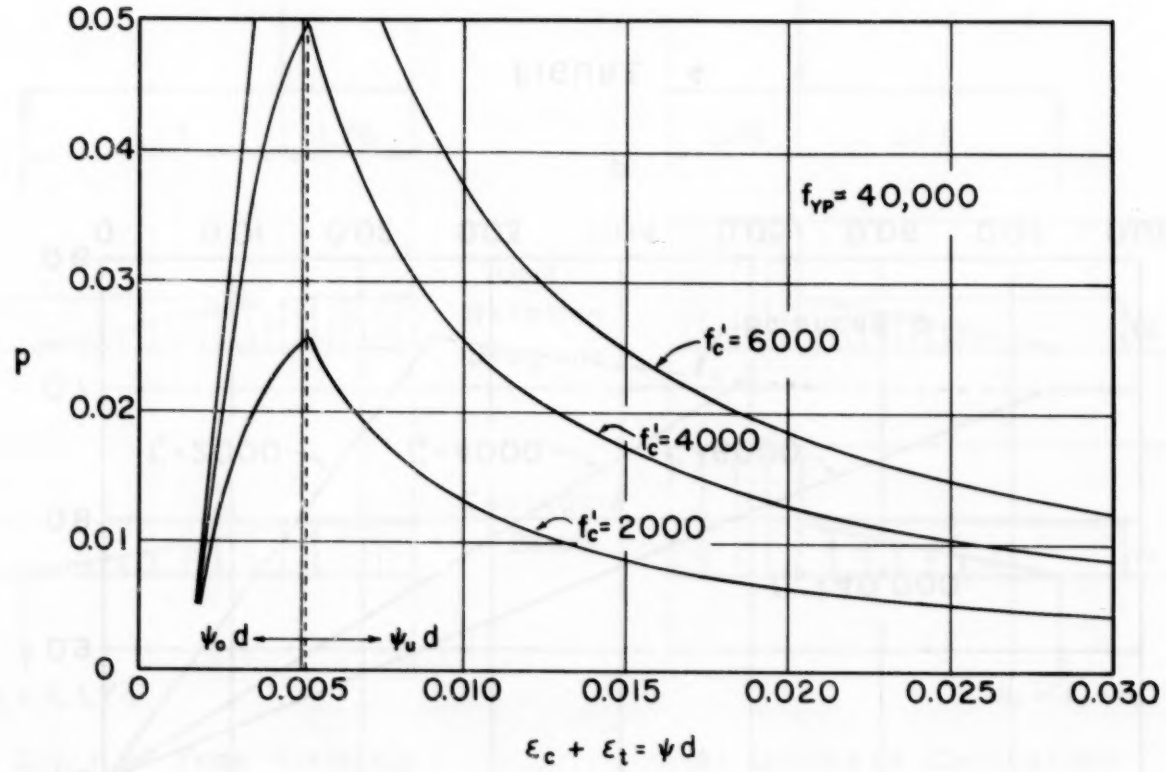

0.05
0.04
0.03
0.02
0.01
0
p
f_{YP}= 40,000
f_c'= 6000
f_c'= 4000
f_c'= 2000
ψ_o d
ψ_u d
0
0.005
0.010
0.015
0.020
0.025
0.030
$\varepsilon_c + \varepsilon_t = \psi$ d

FIGURE 3

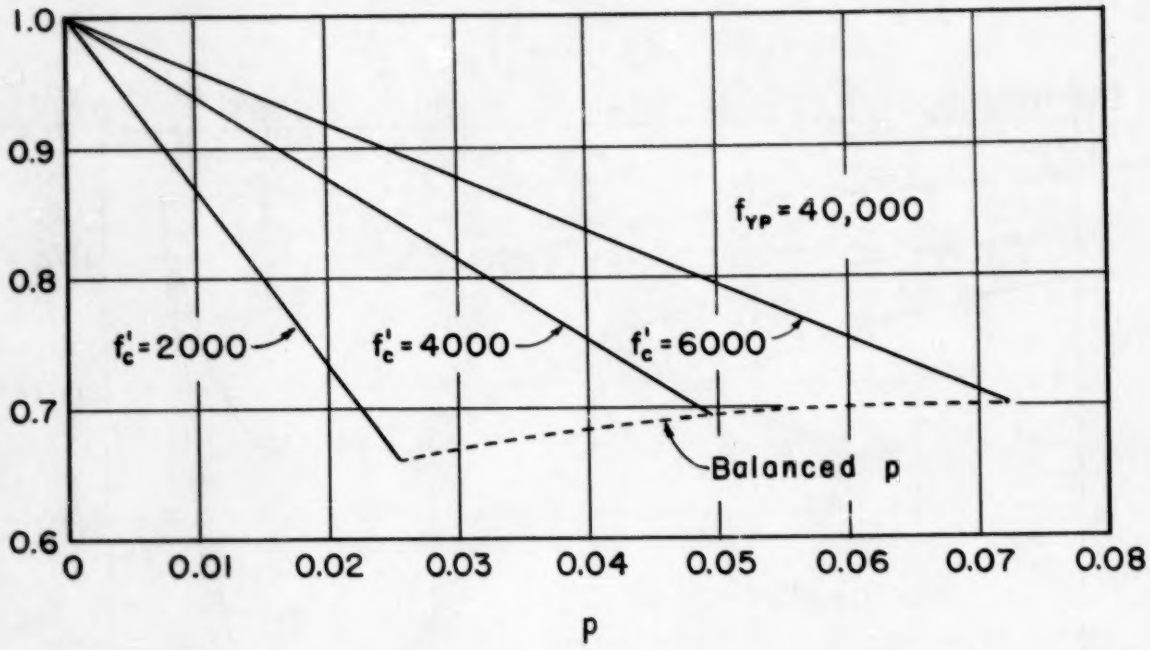
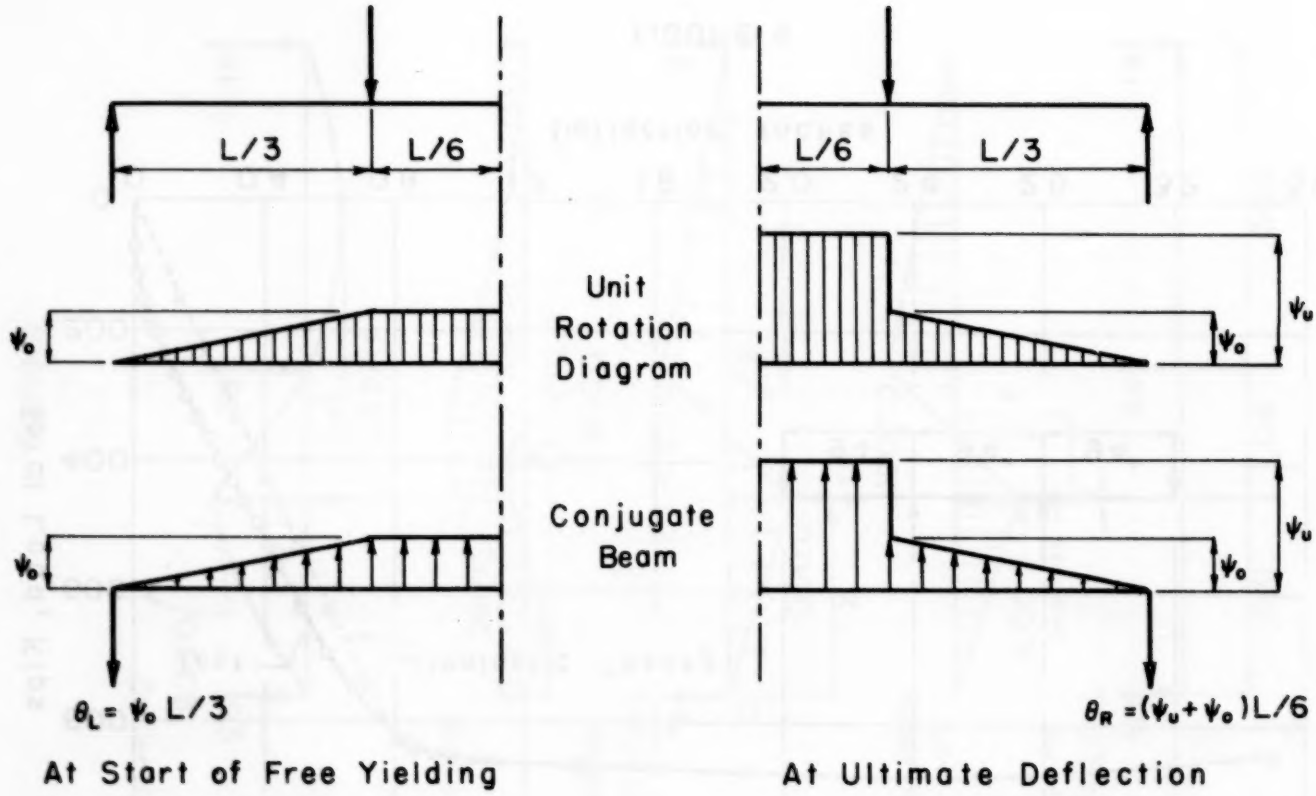

1.0
0.9
0.8
0.7
0.6
j
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
p
f_{YP} = 40,000
f'_c = 2000
f'_c = 4000
f'_c = 6000
Balanced p

FIGURE 4



583-15

FIGURE 5

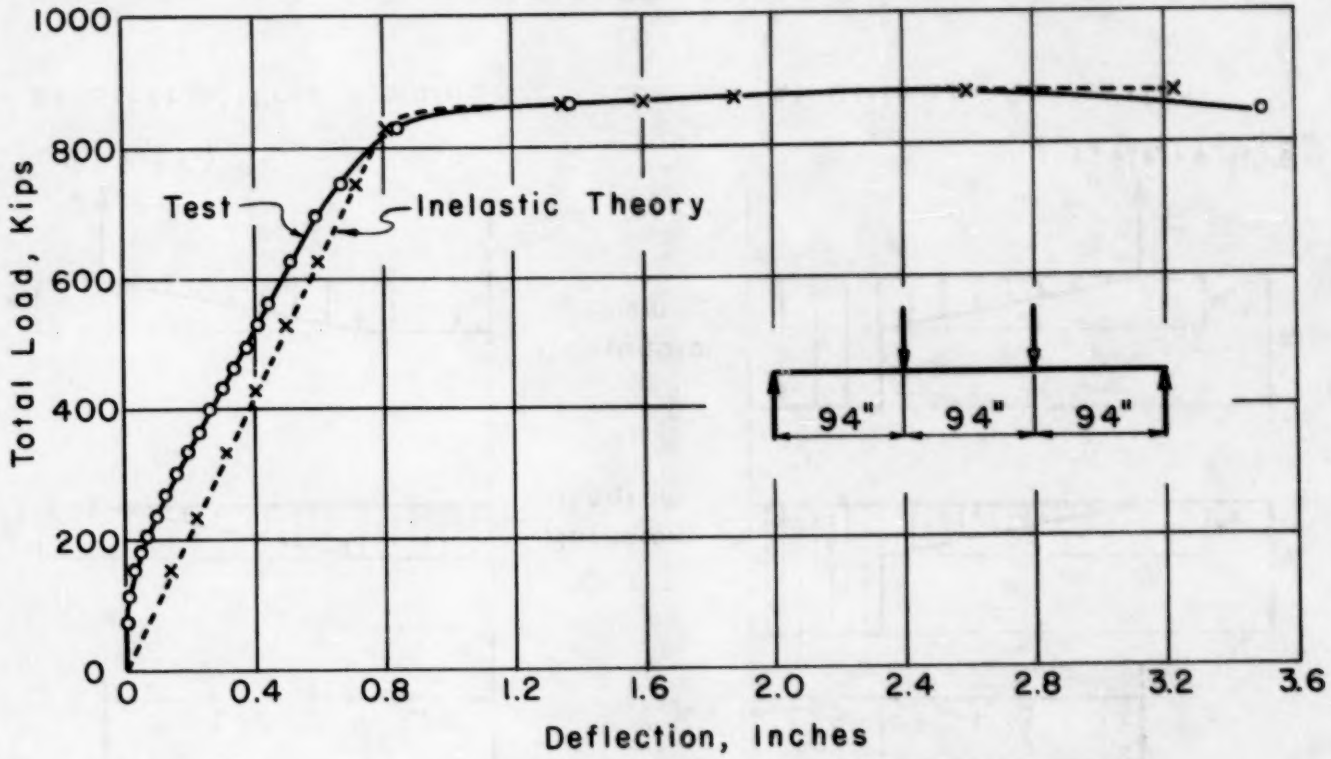


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FIGURE 6

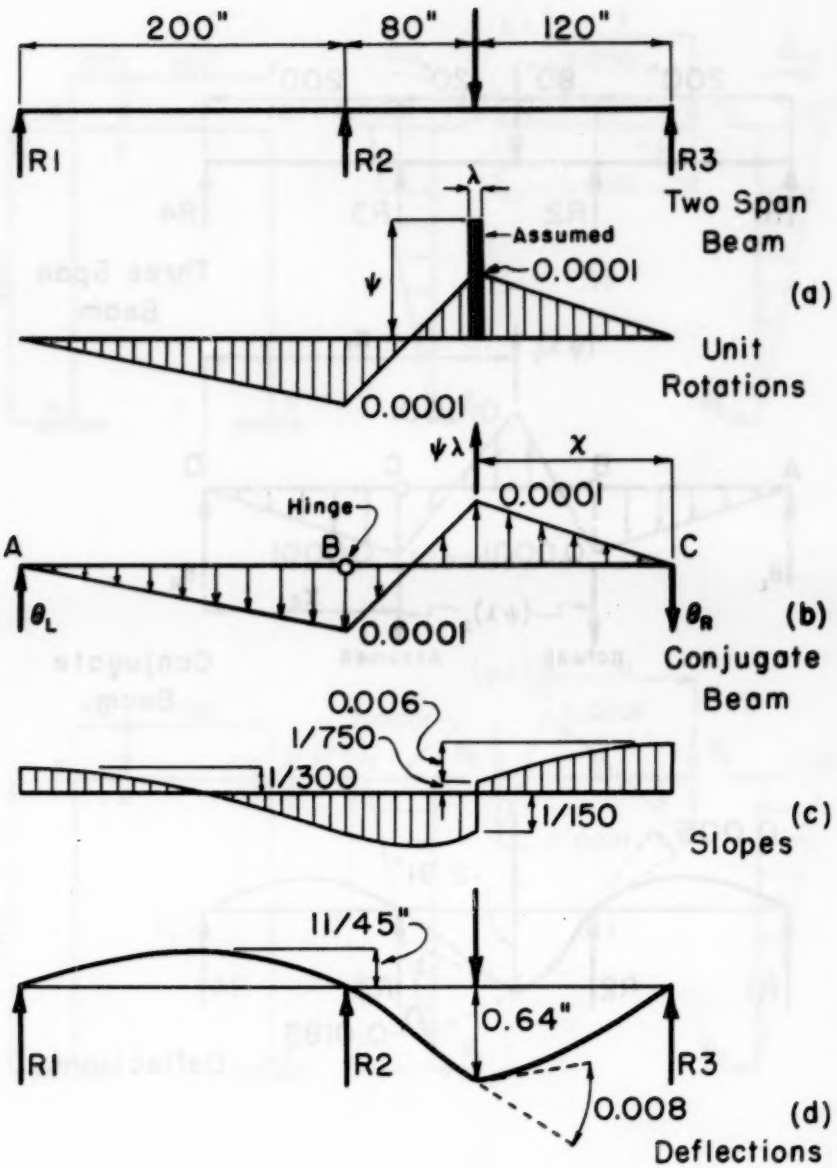
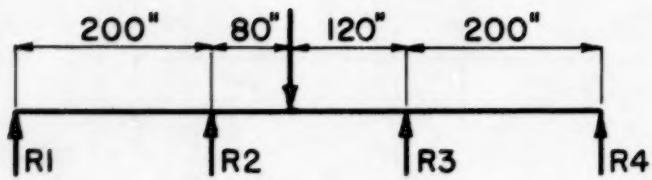
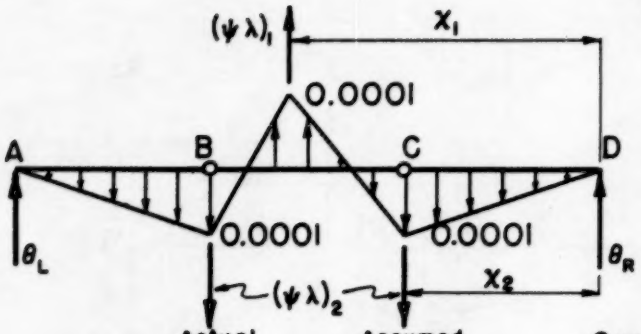

200"
80"
120"
R1
R2
R3
Two Span Beam
λ
Assumed
0.0001
ψ
(a)
Unit Rotations
0.0001
ψλ
χ
0.0001
Hinge
A
B
C
θL
θR
0.0001
(b)
Conjugate Beam
0.006
1/750
1/300
1/150
(c)
Slopes
11/45"
0.64"
R1
R2
R3
0.008
(d)
Deflections

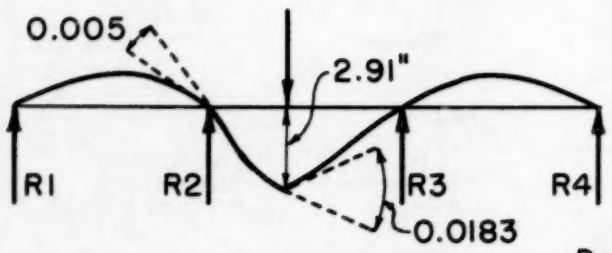
FIGURE 7



Three Span Beam

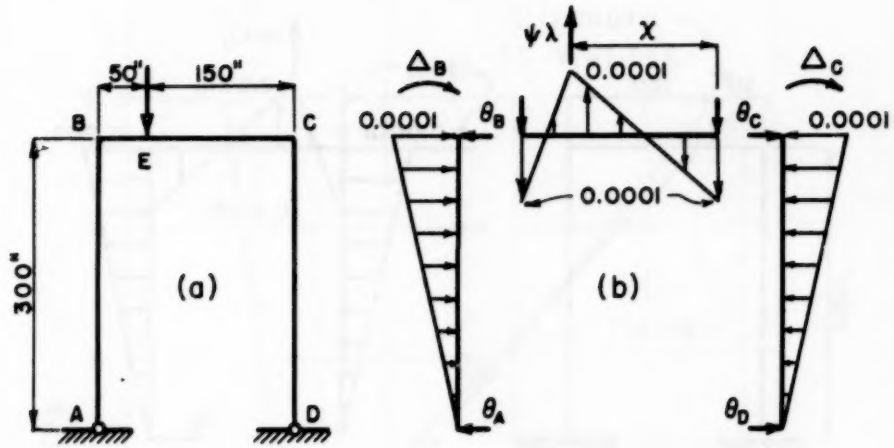


Conjugate Beam

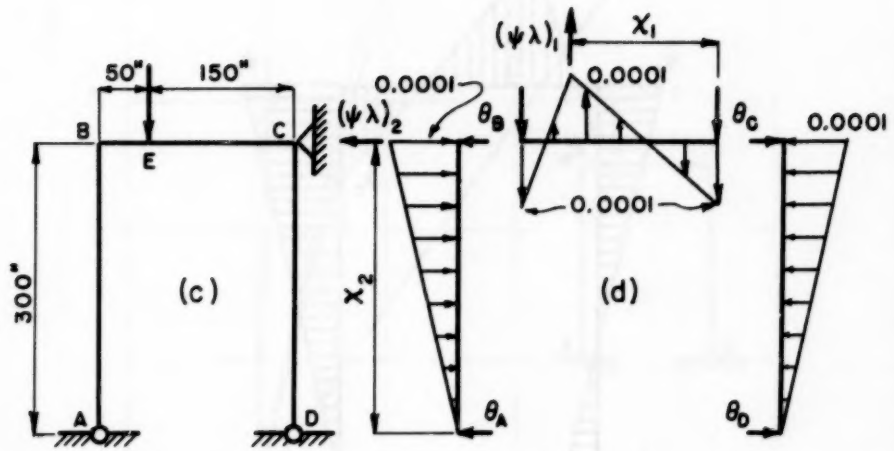


Deflections

FIGURE 8



Free Sidesway



No Sidesway

FIGURE 9

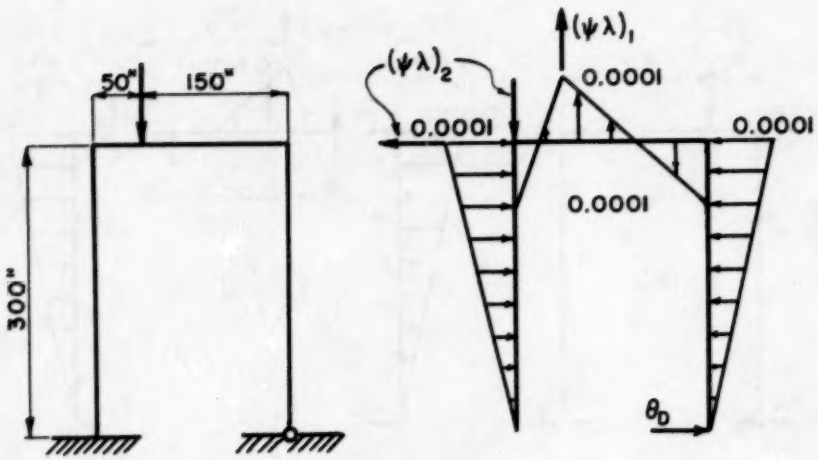


FIGURE 10

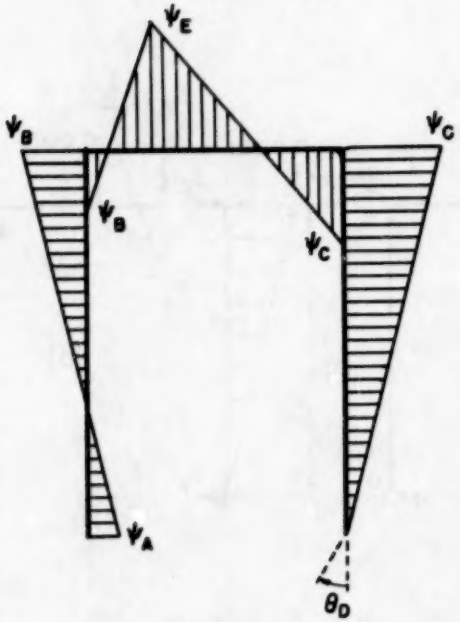


FIGURE 11

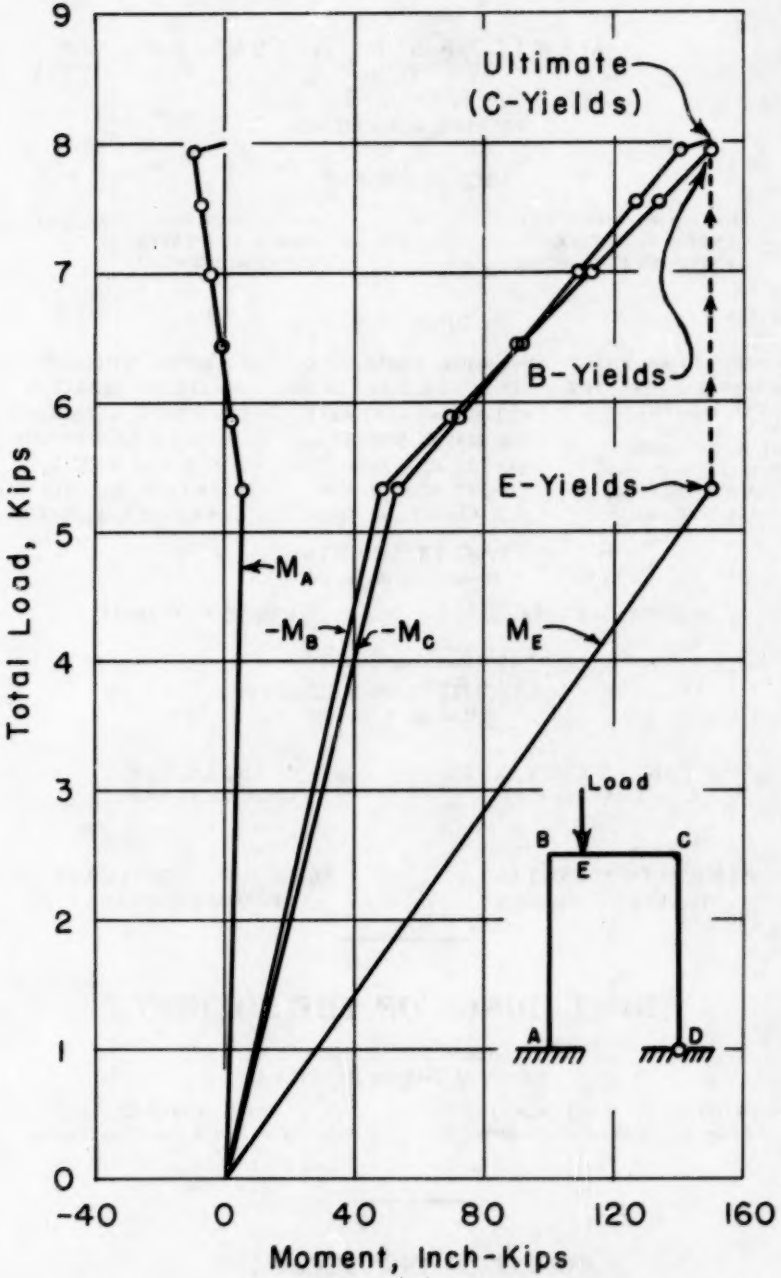


FIGURE 12

583-21

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